
Stochastic covariant calculus with jumps and Stochastic calculus with covariant jumps

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Summary. *We propose a stochastic covariant calculus for càdlàg semimartingales in the tangent bundle TM over a manifold M . In ordinary differential geometry, a connection on M is needed to define the covariant derivative of a C^1 curve in TM ; by applying the transfer principle, Norris has defined a stochastic covariant integration along a continuous semimartingale in TM . We extend this to the case when the semimartingale jumps, using Norris' work and Cohen's results on stochastic calculus with jumps on manifolds. Depending on the order in which the function giving the jumps and the connection are composed, one obtains a "stochastic covariant calculus with jumps" or a "stochastic calculus with covariant jumps", which are in general not equivalent. Under suitable conditions, Norris' results for the continuous case are recovered. This case can be described by a covariant continuous calculus of order two, which involves the notion of a connection of order two.*

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